ON A SIMPLE INTEGRAL REPRESENTATION
OF THE CONTINUOUS $q$-HERMITE POLYNOMIALS $^\dagger$

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Abstract

We derive a simple integral representation and the corresponding Rodrigues type difference formula for the continuous $q$-Hermite polynomials of Rogers. As a consequence, this also yields the appropriate formulae for the Rogers–Szegő and Stieltjes–Wigert polynomials.

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The explicit form of the continuous $q$-Hermite polynomials of Rogers $[13, 3]$ $H_n(\sin \kappa x|q)$ is exhibited by their Fourier expansion

$$H_n(\sin \kappa x|q) = \hat{r}^n \sum_{k=0}^{n} (-1)^k \left[ \begin{array}{c} n \\ k \end{array} \right]_q e^{i(2k-n)\kappa x},$$

where $0 < q = e^{-2\kappa^2} < 1$ and the symbol $\left[ \begin{array}{c} n \\ k \end{array} \right]_q$ stands for the $q$-binomial coefficient (throughout this paper, we will employ the standard notations of the $q$-special functions theory, see [11] or [12]).

The aim of this short paper is to derive a simple integral representation of the continuous $q$-Hermite polynomials $H_n(\sin \kappa x|q)$; furthermore this representation leads to the corresponding Rodrigues type difference formula for $H_n(\sin \kappa x|q)$.

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