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DISTRIBUTED ORDER TIME FRACTIONAL DIFFUSION EQUATION

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on occasion of his 60-th birthday

Abstract

We propose the diffusion-like equation with time fractional derivative of the distributed order for the kinetic description of anomalous diffusion and relaxation phenomena, whose diffusion exponent varies with time and which, correspondingly, can not be viewed as self-affine random processes possessing a unique Hurst exponent. We prove the positivity of the solutions of the proposed equation and establish its relation to the continuous-time random walk theory. We show that the *distributed order time fractional diffusion equation* describes the sub-diffusion random process which is subordinated to the Wiener process and whose diffusion exponent decreases in time (*retarding sub-diffusion*). This process may lead to *ultraslow diffusion*, with the mean squared displacement growing as a power of the *logarithm* of time. We also demonstrate the effect of retarding sub-diffusion by numerical simulation in which the model of Grünwald - Letnikov random walk discrete in space and time is used for simulating a random variable whose probability density evolves in time according to the distributed order time fractional diffusion equation.

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