

A NEW METHOD TO SOLVE BI-OBJECTIVE TRANSPORTATION PROBLEM

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Abstract: In this paper a new method, namely the MMK-method is proposed for finding non-degenerate compromise optimal solution for Bi-objective transportation problem (BTP). The MMK-method derives the set of all possible non-degenerate efficient solutions and it uses the concept of the distance between two points in (x, y) coordinate for finding non-degenerate compromise optimal solution to BTP. A numerical example is given to illustrate the proposed method. A comparative study has also been made between the existing methods and the proposed method.

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Key Words: bi-objective transportation problem, non-degenerate solution, compromise optimal solution

1. Introduction

The classical transportation problem is a special type of network structured linear programming problem which was firstly developed by Hitchcock [2] in 1941. Hitchcock considered his problem as a single objective cost minimizing transportation problem. In real life situation, one can relevantly consider more than one objective in transportation problems. For example, a decision maker wants to minimize total transportation cost simultaneously by minimiz-

ing the total deterioration of the product (in case of perishable or decaying items). Another example of Bi-objective Transportation Problem (BTP) with conflicting objectives comes into light when decision maker would be interested in maximizing the reliability of whole transportation system simultaneously by minimizing the total transportation cost. Many other type of BTP may be formulated by considering other scarce resources such as time of transportation, profit of the system etc.

In literature, essentially, there are many techniques for solving BTP and multiobjective transportation problem proposed by many authors. Bit et al. [1] proposed fuzzy programming technique for solving multicriteria decision making transportation problem. Aneja and Nair [5] presented a method for finding non-dominated extreme points with the help of parametric search in criteria space. Yang and Gen [4] proposed the Evolution program for bi-criteria transportation problem. Pandian and Anuradha [3] presented an algorithm for finding the optimal compromise solution and the set of all efficient and non-efficient solutions to the BTP. But the above [1, 3 and 4] methods do not take care of one of the most important aspect of transportation problem i.e. non-degeneracy of the problem. In this paper, we have proposed a method, namely the MMK-method, for finding the optimal compromise solution to BTP. The concept involved in this method of finding compromise optimal solution is the distance between two points in the coordinate X and Y . The MMK-method seems to be very flexible for decision maker because it derives the set of all non-degenerate feasible solutions. For the sake of validity of the proposed method, a numerical example has been illustrated. A comparison has also been made between the proposed method and various other existing methods. It has also been shown that the proposed method is very lucrative and easy to adopt for obtaining compromise optimal solution to the BTP. The rest of the paper is organised as follows:

In Section 2 a BTP is discussed. In Section 3 a new method named MMK-method is proposed for finding non-degenerate compromise optimal solution of BTP.

2. Bi-objective Transportation Problem (BTP)

Let us consider the following BTP:

$$\text{Minimize } Z_1 = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}$$

$$\text{Minimize } Z_2 = \sum_{i=1}^m \sum_{j=1}^n d_{ij} x_{ij}$$

subject to :

$$\sum_{j=1}^n x_{ij} = a_i, \quad i = 1, 2, \dots, m \quad (1)$$

$$\sum_{i=1}^m x_{ij} = b_j, \quad i = 1, 2, \dots, n \quad (2)$$

$$x_{ij} \geq 0, \quad i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n \quad (3)$$

$$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j \quad (\text{balanced condition}). \quad (4)$$

3. MMK-Method

In this section a new method, named MMK method, is proposed for finding the set of all non-degenerate solutions to BTP and hence the optimal compromise solution also. The stepwise procedure of the MMK method is as follows:

- Step 1: For the given BTP construct two linear programming problems namely FOTP and SOTP as follows

$$\text{Minimize } Z_1 = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}$$

Subject to : *FOTP*
(1 – 3)

and

$$\text{Minimize } Z_2 = \sum_{i=1}^m \sum_{j=1}^n d_{ij} x_{ij}$$

Subject to : *SOTP*
(1 – 3)

- Step 2: Obtain non-degenerate optimal solution to the FOTP as Z_1^* and SOTP as Z_2^* by using ASM method or any other method. Suppose the ideal solution of the BTP is (Z_1^*, Z_2^*) .
- Step 3: Now put all the optimal values of x_{ij} obtained for FOTP in the cost matrix of SOTP as a feasible solution to BTP.
- Step 4: Apply MODI-method to the SOTP obtained from Step 3. And record the values of both objective functions as $(Z_1^{(k)}, Z_2^{(k)})$ at each iteration. Where, k is the number of iterations required for obtaining optimal solution to SOTP.
- Step 5: Record the set $S_1 = \{(Z_1^{(1)}, Z_2^{(1)}), (Z_1^{(2)}, Z_2^{(2)}), \dots, (Z_1^{(k)}, Z_2^{(k)} = Z_2^*)\}$
- Step 6: Now put all the values of x_{ij} obtained for SOTP in the cost matrix of FOTP as a feasible solution to BTP.
- Step 7: Again apply MODI-method to the FOTP obtained from Step 6. And record the values of both objective functions as $(Z_1^{(l)}, Z_2^{(l)})$ at each iteration. Where, l is the number of iterations required for obtaining optimal solution to FOTP.
- Step 8: Record the set $S_2 = \{(Z_1^{(1)}, Z_2^{(1)}), (Z_1^{(2)}, Z_2^{(2)}), \dots, (Z_1^{(l)} = Z_1^*, Z_2^{(l)})\}$.
- Step 9: Combine all the solutions obtained in Step 5 and Step 8 as $S = \{S_1 \cup S_2\}$. Where S is the set of all non-degenerate feasible solutions to BTP.
- Step 10: Now calculate the distance of each point of S from the ideal solution (Z_1^*, Z_2^*) .
- Step 11: From Step 10. identify the point of S for which the distance is minimum. The solution (Z_1^{**}, Z_2^{**}) corresponding to this point would be the compromise non-degenerate optimal solution to BTP.

4. Numerical Illustration

Let us consider the following BTP to illustrate the stepwise procedure of the MMK-method:

	D ₁	D ₂	D ₃	D ₄	Supply
S ₁	(1,4)	(2,4)	(7,3)	(7,4)	8
S ₂	(1,5)	(9,8)	(3,9)	(4,10)	19
S ₃	(8,6)	(9,2)	(4,5)	(6,1)	17
Demand	11	3	14	16	

4.1. Solution using MMK-Method

In the light of Step 1 and Step 2, we have obtained the following FOTP and SOTP:

	D ₁	D ₂	D ₃	D ₄	Supply
S ₁	1	2	7	7	8
S ₂	1	9	3	4	19
S ₃	8	9	4	6	17
Demand	11	3	14	16	

Table 1: FOTP of BTP

Now by using the ASM-method the optimal solution obtained for FOTP is: $x_{11} = 5$, $x_{12} = 3$, $x_{21} = 6$, $x_{24} = 13$, $x_{33} = 14$, $x_{34} = 3$ and the minimum total transportation cost is obtained as 143.

	D ₁	D ₂	D ₃	D ₄	Supply
S ₁	4	4	3	4	8
S ₂	5	8	9	10	19
S ₃	6	2	5	1	17
Demand	11	3	14	16	

Table 2: SOTP of BTP

Now again using the ASM-method, the optimal solution obtained for SOTP is: $x_{13} = 8$, $x_{21} = 11$, $x_{22} = 2$, $x_{23} = 6$, $x_{32} = 1$, $x_{34} = 16$ and the minimum total transportation cost is obtained as 167.

Now the ideal solution of the BTP is: $(Z_1^*, Z_2^*) = (143, 167)$.

Using Step 3, we get the following SOTP with all the solutions of FOTP as its feasible solution.

Now applying Step 4 of the MMK-method, we get the following improved solution at first iteration as follows:

	D ₁	D ₂	D ₃	D ₄	Supply
S ₁	4 ⁵	4 ³	3	4	8
S ₂	5 ⁶	8	9	10 ¹³	19
S ₃	6	2	5 ¹⁴	1 ³	17
Demand	11	3	14	16	

	D ₁	D ₂	D ₃	D ₄	Supply
S ₁	4	4 ³	3 ⁵	4	8
S ₂	5 ¹¹	8	9	10 ⁸	19
S ₃	6	2	5 ⁹	1 ⁸	17
Demand	11	3	14	16	

The value of both objective functions at this solution is $(Z_1^1, Z_2^1) = (168, 215)$. Moving forward iteration by iteration and recording all the values, we get the following set of non-degenerate feasible solutions for BTP:

k	x_{11}	x_{12}	x_{13}	x_{14}	x_{21}	x_{22}	x_{23}	x_{24}	x_{31}	x_{32}	x_{33}	x_{34}	$S_1 = (Z_1^{(k)}, Z_2^{(k)})$
1	5	3	0	0	6	0	0	13	0	0	4	3	(143, 265)
2	0	3	5	0	11	0	0	8	0	0	9	8	(168, 215)
3	0	0	8	0	11	3	0	5	0	0	6	11	(204, 194)
4	0	0	8	0	11	3	5	0	0	0	1	16	(209, 169)
5	0	0	8	0	11	2	6	0	0	1	0	8	(208, 167)

Similarly, by putting all the solutions of SOTP in the cost matrix of FOTP as its feasible solution and following Step 7 and Step 8, we get the following set of non-degenerate feasible solutions as:

l	x_{11}	x_{12}	x_{13}	x_{14}	x_{21}	x_{22}	x_{23}	x_{24}	x_{31}	x_{32}	x_{33}	x_{34}	$S_2 = (Z_1^{(l)}, Z_2^{(l)})$
1	0	0	8	0	11	2	6	0	0	1	0	16	(208, 167)
2	0	2	6	0	11	0	8	0	0	1	0	16	(186, 171)
3	0	3	5	0	11	0	8	0	0	0	1	16	(176, 175)
4	5	3	0	0	6	0	13	0	0	0	1	16	(156, 200)
5	5	3	0	0	6	0	0	13	0	0	14	3	(141, 257)

Following Step 9 and Step 10 the solution set $S = \{S_1 \cup S_2\}$ is obtained and the distance between each point of and the ideal solution $(Z_1^*, Z_2^*) = (143, 167)$ is obtained and tabulated as:

Sl .No.	Objective Values of BTP	Distance from ideal solution $(Z_1^*, Z_2^*) = (143, 167)$
	(143, 265)	98
	(168, 215)	54.012
	(204, 194)	66.70
	(209, 169)	66.03
	(208, 167)	65
	(186, 171)	43.18
	(176, 175)	33.95
	(156, 200)	35.46
	(141, 257)	90.022

Table 3

From the above table it can be seen that the minimum distant solution from the ideal solution is (176, 175) with 33.95 unit distance. So the non-degenerate compromise optimum solution is $(Z_1^{**}, Z_2^{**}) = (176, 175)$.

5. Graphical Representation of Solution

The following Figure 1 represents the set of all non-degenerate optimal solutions of BTP along with the compromise optimal solution.

6. Conclusion

Thus a new method is proposed for finding non-degenerate compromise optimal solution for BTP. The MMK-method provides a non-degenerate compromise optimal solution along with the set of all non-degenerate efficient solutions to BTP. In Table 2 it has been shown that the solution obtained by Aneja and Nair [1] is same as the solution obtained by our proposed method, i.e. (176, 175). Although Bit et al. [2] obtained the solution as (160, 195) which is nearer to the ideal solution (143, 167), and the solution obtained by using the method proposed by Yang and Gen [3] & Pandian and Anuradha [4] are (168, 185) which is more nearer to ideal solution than by Bit et al [2], but all these methods provide degenerate compromise optimal solution. So it is advisable to a decision maker to choose the proposed method because it easily applies and

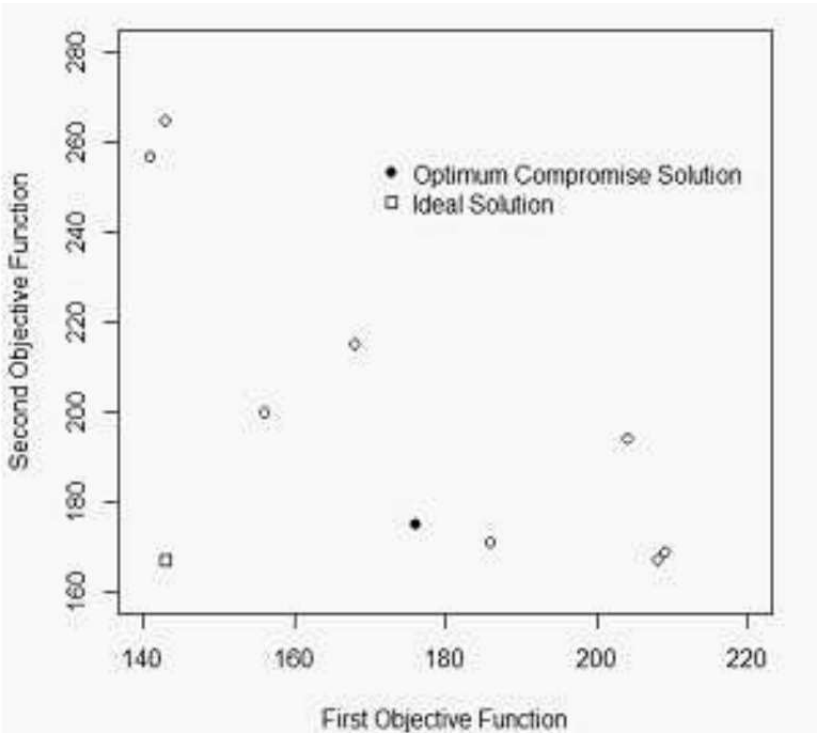


Figure 1

provides non-degenerate compromise optimal solution to BTP and no knowledge of parametric programming is needed to use this method. The summary of the results of the choosen numerical example obtained by the MMK-method and various other existing methods is given in Table 2.

References

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Method Used	Compromise optimal solution	Distance from ideal solution	Nature of the solution
Bit <i>et al.</i> [2]	(160,195)	32.75	Degenerate
Yang and Gen [3]	(168,185)	30.80	Degenerate
Pandian and Anuradha [4]	(168,185)	30.80	Degenerate
Aneja and Nair [1]	(176,175)	33.95	Non-Degenerate
MMK-method	(176,175)	33.95	Non-Degenerate

Table 4

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