International Journal of Applied Mathematics

Volume 27 No. 1 2014, 45-49
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)
doi: http://dx.doi.org/10.12732/ijam.v27i1.5

GLOBAL FEEDBACK CONTROL OF INTERFACE INSTABILITY OF THIN LIQUID FILMS

Emily M. Tian

Department of Mathematics and Statistics
Wright State University
Dayton, OH 45435, USA

Abstract: The control of stationary patterns developed on thin film surface subject to the electric field is studied theoretically near the threshold of instability. The time evolution of the interface between air and polymer film on the unbounded spatial domain is described by the thin film equation; a global feedback control is applied to suppress the subcritical instability. The condition for the controlled pattern to be stable is obtained.

AMS Subject Classification: 35B36, 65P40, 76A20, 76E17
Key Words: thin film, pattern formation, global feedback control, electric field

1. Introduction

The electrohydrodynamic instability plays a critical role in the modern lithographically induced self-assembly process. It guides the creation and replication of lateral structures in polymer films at submicrometer length scales [1, 7, 12, 13]. Understanding the pattern forming instabilities at micro-scales has both technological and scientific interest.

We consider a spatially infinite uniform system near equilibrium where the system may be described by simple equations known as amplitude equations[2]. We are interested in studying the reduced dynamics contained in the amplitude equations near the instability threshold. The conditions for generating certain

Received: December 30, 2013 © 2014 Academic Publications

spatial patterns can be obtained by determining the stability of these amplitude functions.

A prior investigation of the one dimensional evolution process [9] has shown that the amplitude of the linear excitation grows exponentially in a region of ξ values; after that, the exponential growth is arrested by weakly nonlinear effects as the amplitude of linear wave saturates to a finite value dependent on the critical wave number k_c and the thickness ratio ξ. The system becomes unstable to infinitesimal perturbations with wave number k_c. The weakly nonlinear selection mechanism indicates that the periodic finite-amplitude disturbances can entrain waves in a neighborhood of the critical wave number, a field of uniform wave features thus emerge as a consequence of this unique selection. A periodic pattern stationary in time centered around k_c is to arise.The regular finite-amplitude patterns stay on top of the interface in a stationary manner, without visible changes in their shape or speed; the patterns undergo a subcritical stability[10, 11].

Some research was done to explore the possibility to control and suppress the blow up behavior caused by subcritical oscillatory instability in the Ginzburg-Landau equation or Swift-Hohenberg equation, either by introducing locally[4] or globally[3, 8, 6] the control mechanism. In particular, the global feedback control is introduced through a single global parameter that affects the dynamics of the entire system, it can stabilize the unstable modes and produce noval patterns[5]. We attempt to impose the global feedback control to the thin film pattern formation that suffers the similar subcritical instability. Due to the fact that the selection and long term stability of patterns is associated with growing amplitudes, we choose a control based on the maximum of amplitude. We adjust the control parameter to find a satisfactory condition to suppress the subcritical instability.

At the long wavelength limit, we consider the following non-dimensionalized thin film equation[11]:

$$\frac{\partial H}{\partial t} + \alpha\nabla\cdot[H^3\nabla(\nabla^2 H)] + \beta\nabla\cdot[H^3(\epsilon_p\xi + \epsilon_p - (\epsilon_p - 1)H)^{-3}\nabla H] = 0. \quad (1)$$

The unstable initial depth of the uniform thin film grows to reach the next equilibrium state of finite amplitude. Such finite amplitude solutions exhibit subcritical instability which means that they would be stable according to the linearized theory with the damping growth rate, while the weakly nonlinearity incorporates the existence of the patterns. In consideration of the possible cause by a surface tension varying inversely with the strength of the external electric field, we would impose the global feedback control on the coefficient α.

2. Feedback Control of the Thin Film Interface

Near the uniform equilibrium solution $H = 1$, we assume that

$$H = 1 + f,$$

and

$$\alpha(f) = \alpha_0 + p\max_x(|f|), \tag{2}$$

where the control parameter p specifies the feedback intensity.

In the one dimensional setting, we consider

$$f(x,t) = A(t)e^{ikx} + A^*e^{-ikx}, \tag{3}$$

with $|A| = |A^*| = R(t) \ll 1$.

The complex amplitude equation can be obtained after substituting the equations (2) and (3) into the model equation (1):

$$\frac{dA}{dt} + k^4A(1+3R^2)(\alpha_0 + 2pR) - \frac{\beta k^2 A}{(\epsilon_p\xi + 1)^3} = 0. \tag{4}$$

Let $A(t) = R(t)e^{i\theta(t)}$, then the small perturbation f can be eventually reduced by utilizing the powers of its amplitude:

$$\frac{dR}{dt} = (\frac{\beta k^2}{(\epsilon_p\xi + 1)^3} - k^4\alpha_0)R - (2pk^4)R^2 - (3\alpha_0 k^4)R^3 - (6pk^4)R^4. \tag{5}$$

The linear growth rate of the small disturbance f is represented by:

$$\sigma = \frac{\beta k^2}{(\epsilon_p\xi + 1)^3} - k^4\alpha_0,$$

and the symmetry breaking coefficient is:

$$\mu = 2pk^4.$$

If μ is nonzero, namely, when the control p is present, the quadratic term in the amplitude equation dominates the nonlinear evolution of the disturbance f near the instability threshold. For the reason that no stable amplitude solutions can be predicted if μ is $O(1)$, the control parameter p is to be kept small in our concerns, see [2].

In the regime of weakly nonlinearity when the linear growth rate σ is small, the amplitude function $R(t)$ has a long term stationary solution:

$$R_0 = \frac{\sigma}{\mu}.$$

A perturbation around R_0: $R = R_0 + \tilde{R}$ is performed in order to determine the stability conditions of such controlled equilibrium. Introducing $\tilde{R} = e^{\gamma t}$ into equation (5), we find that:

$$\gamma = -\sigma(1 + R_0^2) - \frac{9\alpha_0}{4p^2k^4}\sigma^2. \tag{6}$$

We obtain the following stability condition for R_0 that requires $\gamma < 0$:

$$\sigma > 0 \quad \text{or} \quad \sigma < -\frac{1 + R_0^2}{9\alpha_0 k^4}\mu^2. \tag{7}$$

The conditions (7) suggest that in the case of sub-critically stable steady state, applying the control would stabilize the steady state.

3. Conclusion and Discussions

We have performed the weakly nonlinear analysis in the neighborhood of the stability onset to study the effect of applying the global control of the system of thin film interface evolution driven by the electric field. The global control does provide the stabilization factor of the surface disturbance. The stability condition is obtained.

References

[1] S.Y. Chou, L. Zhuang, Lithographically induced self-assembly of periodic polymer micropillar arrays, *J. Vac. Sci. Technol. B*, **17**, No. 6 (1999), 3197-3202.

[2] M.C. Cross, P.C. Hohenberg, Pattern formation outside of equilibrium, *Review of Modern Physics*, **65**, No. 3 (1993), 851-1112.

[3] A.A. Gloovin, A.A. Nepomnyashchy, Feedback control of subcritical oscillatory instabilities, *Physical Review E*, **73** (2006), 046212.

[4] A. Handel, R.O. Grigoriev, Pattern selection and control via localized feedback, *Physical Review E*, **72** (2005), 066208.

[5] S. Hata, H. Nakao, A.S. Mikhailov, Global feedback control of turing patterns in network-organized activator-inhibitor systems, *EPL*, **98** (2012), 64004.

[6] Y. Kanevsky, A.A. Nepomnyashchy, Interaction of solitary waves governed by a controlled subcritical Ginzburg-Landau equation, *PIERS Online*, **3**, No. 2 (2007), 154-157.

[7] E. Schäffer, T. Thurn-Albrecht, T.P. Russell, U. Stelner, Electrically induced structure formation and pattern transfer, *Nature*, **403** (2000), 874-877.

[8] L.G. Stanton, A.A. Golovin, Global feedback control for pattern-forming systems, *Physical Review E*, **76** (2007), 036210.

[9] E.M. Tian, Pattern formation induced by an electric field in thin liquid films, *Journal of Mathematics, Statistics, and Allied Fields*, **1**, No. 1 (2007), 1-6.

[10] E.M. Tian, T.P. Svobodny, J.D. Phillips, Stability of thin film patterns induced by electro-static field, *International Journal of Applied Mathematics*, **24**, No. 2 (2011), 221-228.

[11] E.M. Tian, T.P. Svobodny, J.D. Phillips, Thin liquid film morphology driven by electro-static field, *Appl. Math. Mech.-Engl. Ed.*, **32**, No. 8 (2011), 1039-1046.

[12] N. Wu, W. Russel, Electric-field-induced patterns in thin polymer films: Weakly nonlinear and fully nonlinear evolution, *Langmuir*, **21** (2005), 12290-12302.

[13] N. Wu, W.B. Russel, Micro- and nano-patterns created via electrohydrodynamic instabilities, *Nano Today*, **4** (2009), 180-192.