ANALYSIS OF THE STRUCTURAL INTEGRITY OF A BUILDING BY COMPLEX WAVELETS

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Abstract: In this paper, the analysis of the structural integrity of a building by complex wavelets is performed. Complex wavelet provides desirable properties to fault identification like shift invariance and good directional selectivity. In order to perform numerical experiments, mathematical modeling of a building of two walks is performed. The database is built through simulations of different situations: base-line condition and improper conditions. Simulation results showed that complex wavelets achieved good performance in the problem studied.

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1. Introduction

The analysis of structural integrity is an important field of research from the
economic and security viewpoint. Due to inappropriate use of materials, lack of care in the project execution and even lack of maintenance, there is a large amount of newly constructed buildings presenting failures. In this sense, the development of new techniques for digital signal processing is very important to help engineers and technicians to identify faults in structures such as buildings, see [7].

More earlier works in analysis of structural integrity are based on nondestructive testing as in [7]. For this purpose, the wavelet analysis is a powerful tool for signals analysis. Wavelet analysis provides a representation process of a signal by an orthonormal basis of functions that oscillate locally. This base performs a time-frequency decomposition of the analyzed signal by means of stretched and shifted versions of a fundamental function $\psi$ as follows:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}} \psi \left( \frac{t-b}{a} \right).$$

In (1), $\psi$ is named mother wavelet, whereas $a$ and $b$ are the scale and shift factors, respectively. The wavelet coefficients can be evaluated from $\langle x, \psi_{a,b}^* \rangle$, where $\psi_{a,b}^*$ is the complex conjugate of the wavelet $\psi_{a,b}$ and $x(t)$ is the input signal.

In practice, the wavelet coefficients $d(j,n)$ are obtained by the discrete wavelet transform (DWT). The DWT has the ability to localize singularities through wavelets scales $j$ ($j = 1, 2, 3...$). It is possible because $d(j,n)$ correspond to the frequency situated approximately between $(2^{-j}f_s, 2^{-j-1}f_s)$, where $f_s$ is the sampling rate of the input signal $x(t)$ [6].

In spite of the extensive use of the DWT in structural integrity analysis, for example in [3], in [1] the authors showed that dual-tree complex wavelet transform (dual-tree CWT) is better than DWT in small flaws identification. In this sense, the authors have used complex wavelets in the problem of fault monitoring and identification in aeronautical structures.

Unlike the normal DWT, where $\psi$ is real, in the dual-tree CWT $\psi$ is complex and its real and imaginary parts are computed by two real filter banks. In other words, for the implementation of the dual-tree CWT two real DWTs are necessary. In order to achieve some desirable properties for signal processing, like shift invariance, the filter sets should be designed taking into account some additional requirements to the DWT. Details about filters design can be found in [5].

This paper shows the advantages in using complex wavelets for analysis of the structural integrity of a building. For this purpose, the methodology presented in [1] will be employed. The objective is to verify if complex wavelet
is suitable for the present problem too. The remainder of the paper is organized as follows: Modeling and simulations are presented in Section 2, analysis based on complex wavelet is presented in Section 3. Finally, concluding remarks are presented in Section 4.

2. Modeling and Simulations

In order to perform numerical experiments, a mathematical model using differential equations representing a two-story building with two degrees of freedom each was developed. Figure 1 illustrates the structure model employed.

The building is an adimensional model and consists of two masses ($M_1$ and $M_2$), with stiffness coefficients ($K_1$ and $K_2$) and elasticity ($C_1$ and $C_2$). From Hamilton’s principle, applies the Lagrange function adopting the coordinates $q_1$ and $q_2$ (see Figure 1). Therefore, it follows that:

$$L = \frac{1}{2} \left[ M_1 (\ddot{q}_1 + \dot{q}_1)^2 + M_2 (\ddot{q}_2 + \dot{q}_2)^2 - (K_1 q_1^2 + K_2 (q_2 - q_1)^2) \right]. \quad (2)$$

From (2), using the Lagrange function for generalized coordinates $q_1$ and $q_2$, it follows that

$$\begin{align*}
M_1 \ddot{q}_1 + K_1 q_1 - K_2 (q_2 - q_1) + C_1 \dot{q}_1 &= -M_1 \ddot{S}, \\
M_2 \ddot{q}_2 + K_2 (q_2 - q_1) + C_2 (\dot{q}_2 - \dot{q}_1) &= -M_2 \ddot{S}. 
\end{align*} \quad (3)$$

Dividing the first and second equations by $M_1$ and $M_2$ respectively:

$$\begin{align*}
\ddot{q}_1 + \frac{\omega_1^2}{M_1} q_1 - \frac{K_2}{M_1} q_2 + \frac{C_1}{M_1} \dot{q}_1 &= -\ddot{S}, \\
\ddot{q}_2 + \frac{\omega_2^2}{M_2} (q_2 - q_1) + \frac{C_2}{M_2} (\dot{q}_2 - \dot{q}_1) &= -\ddot{S}. 
\end{align*} \quad (4)$$
where $\omega_1^2 = (K_1 + K_2)/M_1$ and $\omega_2^2 = K_2/M_1$ are the natural frequencies and $-\ddot{S}$ is the external excitation (natural forces).

Transforming the system in (4) to state space equations where $x_1 = q_1$, $x_2 = \dot{q}_1$, $x_3 = q_2$ and $x_4 = \dot{q}_2$, it follows that

$$
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\omega_1^2 x_1 + \frac{K_2}{M_1} x_3 - \frac{C_1}{M_1} x_2 - \ddot{S} \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \omega_2^2 (x_1 - x_3) + \frac{C_2}{M_2} (x_2 - x_4) - \ddot{S}.
\end{align*}
$$

Simulations consisted 1400 signals in the structure, 400 in base-line condition and 1000 with structural faults. The values of the parameters adopted for numerical simulations were $\omega_1 = 1$, $M_1 = 7$, $C_1 = 0.6$, $K_2 = 0.8$, $\omega_2 = 2$, $C_2 = 0.7$, $M_2 = 7$ to the structural system in [2]. Faults were simulated from several changes (gains) in mass $M_1$. The excitation signal used was:

$$
S(t) = \exp(-5 \cos(t)).
$$

The database is formed by velocity and displacement signals of the excited structure.

3. Analysis of the Structural Integrity of a Building based on Complex Wavelets

As previously mentioned, in the conventional DWT the function $\psi$ in (1) is real, whereas in the dual-tree CWT it is complex, generating complex wavelets coefficients as follows:

$$
d_c(j, n) = d_r(j, n) + j d_i(j, n). \tag{7}
$$

The real and imaginary parts, $d_r(j, n)$ and $d_i(j, n)$ respectively, are calculated individually by two separate DWT filter banks [8].

The proposed method in [1], consists in a multiscale analysis of the phase of the complex wavelet coefficients in (7). It was shown in [1] that the phase is very sensitive to variations of the signal, obtaining a good performance in small flaws identifications. In this sense, scales $j$ ($j = 1, 2, 3, 4$) are analyzed and the phase is obtained from (7) as follows:

$$
\angle d_c(j, n) = \arctan \left( \frac{d_i(j, n)}{d_r(j, n)} \right). \tag{8}
$$
In order to identify a fault, Pearson's correlation coefficient will be calculated on (8), [1]. Pearson's correlation coefficient measures the mutual relationship between base-line signal and the failure signal. Correlation coefficient equal to 1 or -1 implies a perfect correlation, whereas correlation coefficient equal to 0 means that the signals are not linear correlated [4]. In this sense, as close to zero is the correlation coefficient as better is the fault detection [1].

The analysis of structural integrity of the modeled building was performed as follows: First, 210 signals in base line condition was used to define a threshold from Pearson's correlation coefficient. This data set corresponds to 15% of the available data. Second step consists in presenting the rest of the signals randomly and checking, from phase and Pearson's correlation coefficient, which are faulty.

Using the threshold empirically defined from Pearson's correlation coefficient in the first step, it was found a success rate of 100% in the fault characterization. It is noteworthy that the phase is uncorrelated with the energy of the signal in time domain. This fact is very useful in structural fault identification. All results were obtained by using Antonini (9,7)-tap filters at first level and q-shift (10,10)-tap filters at level $j > 1$ in the dual-tree CWT [5].

4. Conclusion

In this paper, the analysis of the structural integrity of a building by complex wavelets was evaluated. In order to implement a complex wavelet transform, the dual-tree architecture was employed. The dual-tree CWT have some desirable properties for signal processing, as nearly shift-invariance, overcoming the DWT in this aspect.

The phase of the complex wavelet coefficients showed to be very sensitive to variations of the signal analyzed, capturing small flaws. For the application studied, complex wavelet showed to be a powerful tool to fault identification.

Numerical experiments from a mathematical model that represents a building of two walk was done. In simulations, there were used signals with several structural faults and the response of the system on a certain excitation was obtained. Simulation results showed that the proposed method presented good results, obtaining a hit rate of 100 % in fault detection.

References


